

## DOCUMENT RESUME

ED 338 500

SE 052 356

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TITLE Error Patterns in Problem Solving.  
PUB DATE 6 Oct 90  
NOTE 13p.; Paper presented at the International Conference of the Council for Learning Disabilities (12th, Austin, TX, October 6, 1990).  
PUB TYPE Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.  
DESCRIPTORS Arithmetic; \*Cognitive Development; Computation; Elementary School Mathematics; \*Error Patterns; Instructional Improvement; \*Instructional Leadership; Intermediate Grades; \*Learning Disabilities; Mathematical Concepts; Mathematics Education; \*Mathematics Skills; Mathematics Teachers; \*Problem Solving; Whole Numbers; Word Problems (Mathematics)

IDENTIFIERS \*Error Analysis (Mathematics)

## ABSTRACT

Although many common problem-solving errors within the realm of school mathematics have been previously identified, a compilation of such errors is not readily available within learning disabilities textbooks, mathematics education texts, or teacher's manuals for school mathematics texts. Using data on error frequencies drawn from both the Fourth National Assessment of Educational Progress and a study of the problem-solving performances of 431 fifth- and sixth-grade students from southern California, error patterns and their probable sources for traditional one-step word problems employing whole numbers can be described. Problem-solving errors are divided into four broad categories: (1) computational errors; (2) operational errors; (3) non-attempt errors; and (4) miscellaneous errors. Teacher knowledge of error patterns in problem solving promotes positive outcomes, yet teachers are often lacking in knowledge of error pattern analyses, particularly teachers of students with learning disabilities. The findings indicate not only that appropriate student-error analysis leads to accurate feedback on the source of student errors, but also that an understanding of error patterns can reveal underlying conceptual misunderstandings and/or lack of proper problem-solving strategies on the part of the students; such findings can ultimately become the focus for further instructional planning. (JJK)

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ERROR PATTERNS  
IN  
PROBLEM SOLVING

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OCTOBER 6, 1990

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ABSTRACT

ERROR PATTERNS IN PROBLEM SOLVING

This paper describes error patterns and their probable sources for traditional one-step word problems utilizing whole numbers. Teachers of students with learning disabilities often are lacking in knowledge of error pattern analysis in mathematical problem solving. Although many common problem-solving errors have been identified, a compilation of errors is not readily available in learning disabilities texts, mathematics education texts or in teacher's manuals of student mathematics texts.

Problem-solving errors are divided into four broad categories: computation errors, operational errors, miscellaneous errors, and no-attempt errors. Error frequencies are drawn from the Fourth National Assessment of Educational Progress and the author's study of the problem solving performance of 431 fifth- and sixth-grade students from southern California.

Teacher knowledge of error patterns in problem solving should have several positive outcomes. First, it should lead to accurate feedback to students on the source of their errors. Secondly, error patterns often reveal underlying misunderstandings of mathematical concepts, lack of problem-solving strategies, and/or immature problem solving strategies that can then become the focus of instructional planning.

## Error Patterns in Problem Solving

The process of analyzing error patterns in computation (Ashlock, 1976; Brown & Burton, 1978) has proven beneficial to teachers and students. Through this analysis teachers are made aware that student errors are often systematic. Computation error analysis has also highlighted the many different routes students use to arrive at wrong answers. The positive outcome has been adapted instruction. As teachers recognize error patterns, they are able to revise instruction to challenge the misconceptions which often lead to computation errors.

Less attention has been directed to error pattern analysis in traditional problem solving. Although many common problem-solving errors have been identified, a compilation of errors is not readily available in mathematics education texts or in teacher's manuals of student mathematics texts. Most preservice teachers begin teaching careers without knowledge of problem-solving errors and inservice teachers rely on experience over time to rectify this omission.

Teacher knowledge of error patterns in problem solving should have several positive outcomes. First, it could lead to more accurate feedback to students on the type and source of their errors. Secondly, error patterns often reveal underlying misunderstandings of mathematical concepts, lack of problem-solving strategies and/or immature problem-solving strategies (Sowder, 1988) that can then become the focus of instructional planning.

This article will describe error patterns and their probable sources for traditional one-step word problems utilizing multi-digit whole numbers similar to those usually found in fifth- and sixth-grade mathematics texts. Identification of error frequencies will be drawn from the Fourth National Assessment of Educational Progress (Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988) and the author's study of the problem solving performance of 431 fifth- and sixth-grade students from southern California (Babbitt, 1986).

Problem-solving errors will be divided into four broad categories: computation errors, operational errors, miscellaneous errors, and no-attempt errors (Fig. 1). In some instances, both the nature of the error and its probable source can be revealed by students' written work. In many instances, further probing through student interviews is necessary to determine the

student's solution process (Liedtke, 1988). In addition to describing the written errors, the discussion that follows will highlight the most common misconceptions which underlie problem-solving errors.

#### Computation Errors

The most frequent problem-solving errors are in computation (Kouba et al., 1988; Muth, 1984; Suydam & Riedesel, 1969). For example, 13-18% of fifth-and sixth-grade students make computation errors on three-digit subtraction when such computation is required to solve a word problem (Babbitt, 1986). This remains at 15% for seventh grade students (Kouba et al., 1988). Over 30% of fifth-and sixth-grade students make computation errors on three-digit by two-digit multiplication and four-digit by two-digit division. Kouba et al. (1988) reported that multi-digit multiplication errors dropped to 23% for seventh graders on problems which required no regrouping. Computation errors significantly influence problem-solving performance when multi-digit numbers are used.

#### Operational Errors

Operational errors are a second major source of problem-solving errors (Bell, Costello, & Kucheman, 1983). Approximately half the errors of the fifth and sixth graders on multiplication and division word problems were due to incorrect choice of operation. Fifth and sixth graders made four types of operational errors: inverse operations, simpler operations, random choices, and use of immature strategies.

Students frequently choose the inverse operation; that is, they make the error of subtracting when they should add or multiplying when they should divide or vice versa. Some students choose a simpler operation than that required by the problem. For multiplication, the incorrect operation most frequently chosen is addition. At times, the choice of operation appears to be random. For some division problems, incorrect operations are equally distributed among addition, subtraction, and multiplication. Immature strategies in which the student chooses an operation based upon key words, the size of numbers, or the number of numbers, are used by many students (Sowder, 1988). Students will add or multiply if the two numbers have about the same number of digits. If one number has more digits than the other, they will divide.

Large numbers of students have difficulty deciding which operation to use to solve a problem.

### Miscellaneous Errors

Students will sometimes choose the correct operation, but still fail to solve a word problem correctly. The fault usually lies in the information that is used and in the way it is used in setting up the problem computation. These miscellaneous errors include extraneous-information errors, order-of-language errors, and other unique errors.

#### Extraneous-Information Errors

Using extraneous information is a common problem-solving error. Kouba et al. (1988) reported 26% of third graders and 16% of seventh graders used extraneous information in the solution of an addition problem in which three numbers were given but only two were necessary. Twenty percent of fifth and sixth graders used extraneous information in the solution of a similar problem (Babbitt, 1986). Sowder (1988) included this error under immature strategies. It has been included here under miscellaneous errors because the correct operation was chosen.

#### Order-of-Language Errors

The order of problem language influences errors. Students often set up the problem in the order of the numbers presented. While this works for addition and multiplication which are commutative, it leads to errors for subtraction and division. Six percent of the students committed this error. The strategy becomes totally unreliable on two-step problems.

#### Unique Errors

Sometimes unique errors are revealed in students' solutions. In the author's study, fifth- and sixth-grade students in a single school used the remainder as the answer to a division problem with greater frequency than similar students in other schools. In Figure 2, students first made computation errors to produce remainders and then used the remainder as the solution. Interestingly, the error was revealed because students were required to transfer their answers to a designated answer space. Unique errors are only limited by student ingenuity and instructional inadequacy.

### No-Attempt Errors

Upper elementary students attempt most whole number problems but failure to attempt problems increases as the problems become more difficult. Kouba et al. (1988) did not report the frequency of no-attempt errors. In my own study, problems which were solved correctly by 90% of the fifth- and sixth-grade students were attempted by over 99% of them. A division problem which was solved correctly by only 32% of the students was attempted by 91%. Fifth and sixth graders attempt most addition and subtraction word problems requiring written computation. A few no-attempts are present for multiplication (1.4 - 2.8%) but division is the operation which students avoid most frequently (2.6 - 9.0% no-attempts).

Student interviews provide the best means to explore the source of no-attempts. Avenues of exploration include student ability to read the problem, student avoidance due to computation complexity or operational uncertainty, and lack of sufficient time to solve the problem. The source of no-attempt errors should guide future instruction. It has been shown that no-attempts are reduced when low achieving students are allowed to use calculators when solving word problems (Babbitt, 1986).

### Implications for Instruction

Four major student errors in mathematical problem solving have been identified. Balacheff (1990) hypothesized that errors are not just failures, but are symptoms of specific pupils' conceptions. In accordance with this view, each error and its subtypes may require a specific instructional strategy on the part of the teacher. Some suggested instructional strategies which are linked to specific error patterns will be described.

Accurate operational choices accompanied by a preponderance of computation errors may suggest the use of calculators to support the problem-solving process. In this first case, the student shows understanding of the problem situation but needs assistance with carrying out the computation. In contrast, operational errors often reflect a lack of understanding of the problem situation. For example, inverse-operation errors, or random-operation errors, may require the teacher to clarify the problem-solving situation through concrete models, visual depictions and role-playing, when appropriate. Choice of simpler operations may mean the

student is working within a limited conceptual framework. For example, the student may understand that both addition and multiplication can be used to attain a total, but may need additional instruction in when and how each operation is used. Simpler operations are also sometimes used when students are unable to compute using the more complex operation. Here, again, the use of a calculator may alleviate the difficulty. Use of immature strategies usually means the student is solving problems without reference to math concepts. The student perceives achieving an answer as the important task, rather than understanding the problem. If students are using immature strategies, the teacher needs to change the focus of instruction to thinking strategies which are rewarded more highly than answer production.

The third error type, miscellaneous errors, require additional instructional strategies. Language-order errors require changing the types of problems studied so that a variety of problems are solved in which expected and unexpected order are incorporated. Instruction is designed to break the patterned thinking of the student. Use of extraneous information suggests the need for activities in which deciding what is needed to solve problems and gathering data to solve problems is emphasized. Unique errors obviously require careful analysis before instructional strategies can be suggested.

Fourth, no-attempt errors also require further gathering of information since the source of the errors can be so varied. The student's ability to read the problems should be checked. Diagnostic teaching can be used to explore computational or operational roadblocks to adequate problem solving. The misunderstandings uncovered will probably fall into the other three major categories described and previously suggested strategies can then be used.

### Conclusion

Problem-solving error analysis can be used to improve instruction and subsequent student learning. It is naive to assume that students will make no errors or that an enhanced problem-solving curriculum as suggested in the NCTM curriculum standards (1989) will prevent errors entirely. While expert teachers are already able to categorize problems to be solved at high cognitive levels (Berliner, 1986), problem-solving error analysis helps novice teachers organize the information they are gathering on students' problem solving into coherent patterns which can then be used to inform teacher decisions about instruction.

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# PROBLEM-SOLVING ERRORS

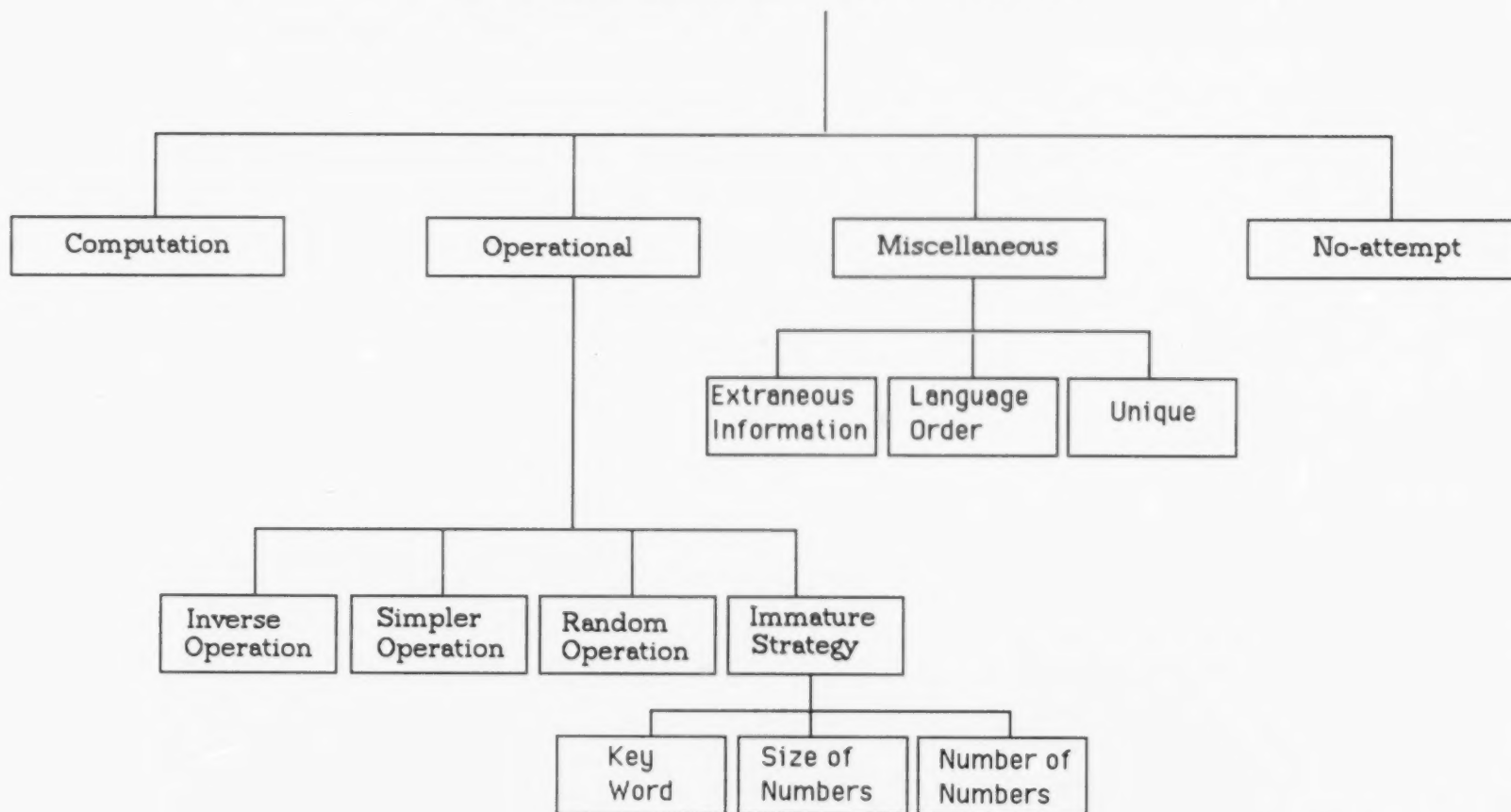


Figure 1

# UNIQUE ERROR

13. The bakery sent 2158 packages of rolls to grocery stores. 83 trays were filled with rolls. How many packages were on each tray?

**34** Incorrect response

$$\begin{array}{r} 28 \\ 83 \overline{) 2158} \\ \underline{146} \\ 698 \\ \underline{664} \\ 34 \end{array}$$

**26** Correct response

$$\begin{array}{r} 26 \\ 83 \overline{) 2158} \\ \underline{166} \\ 498 \\ \underline{498} \end{array}$$

Figure 2. Example of unique error: Remainder used as solution to division problem.